CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Draw curve with increasing gradient existing for negative and positive values of x	**M1**	
	Draw correct curve passing through the origin	**A1**	[2]
2	Either: State correct unsimplified x^2 or x^3 term	**M1**	
	Obtain $a = -9$	**A1**	
	Obtain $b = 45$	**A1**	
	Or: Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$	**M1**	
	Obtain $f''(x) = -18(1+9x)^{-\frac{5}{3}}$ and hence $a = -9$	**A1**	
	Obtain $f'''(x) = 270(1+9x)^{-\frac{8}{3}}$ and hence $b = 45$	**A1**	[3]
3	Use correct quotient rule or equivalent to find first derivative	**M1***	
	Obtain $\dfrac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
	Substitute $x = \frac{1}{4}\pi$ to find gradient	**dep M1***	
	Obtain $-\frac{3}{2}$	**A1**	
	Form equation of tangent at $x = \frac{1}{4}\pi$	**M1**	
	Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent	**A1**	[6]
4	**(i)** Use $\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}$ and equate $\dfrac{dy}{dx}$ to 4	**M1**	
	Obtain $\dfrac{4p^3}{2p+3} = 4$ or equivalent	**A1**	
	Confirm given result $p = \sqrt[3]{2p+3}$ correctly	**A1**	[3]
	(ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0	**M1**	
	Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively)	**A1**	[2]
	(iii) Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$	**M1**	
	Obtain final answer 1.89	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895)	**A1**	[3]

5	State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent		**B1**	
	Use identity $\sin 2x = 2\sin x\cos x$		**B1**	
	Carry out complete substitution, for x and $\mathrm{d}x$		**M1**	
	Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent		**A1**	
	Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$		**M1***	
	Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$		**A1**	
	Apply correct limits correctly		**dep M1***	
	Obtain $\frac{20}{3}$ or exact equivalent		**A1**	[8]
6	State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$		**B1**	
	Divide by $\cos A$ to find value of $\tan A$		**M1**	
	Obtain $\tan A = 3$		**A1**	
	Use identity $\sec^2 B = 1 + \tan^2 B$		**B1**	
	Solve three-term quadratic equation and find $\tan B$		**M1**	
	Obtain $\tan B = \frac{3}{2}$ only		**A1**	
	Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$		**M1**	
	Obtain $\frac{3}{11}$		**A1**	[8]
7	**(i)**	<u>Either</u> Substitute $x = -1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
		<u>Or</u> Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
	(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
		Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
		State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$	**A1**	
		Use relevant method for finding at least one constant	**M1**	
		Obtain one of $A = -2, B = 1, C = 8$	**A1**	
		Obtain all three values	**A1**	
		Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
		Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of … $+ c$	**A1**	[8]

8	**(i)**	Express a general point on the line in single component form, e.g. $(\lambda, 2-3\lambda, -8+4\lambda)$, substitute in equation of plane and solve for λ		**M1**	
		Obtain $\lambda = 3$		**A1**	
		Obtain $(3, -7, 4)$		**A1**	[3]
	(ii)	State or imply normal vector to plane is 4**i** – **j** + 5**k**		**B1**	
		Carry out process for evaluating scalar product of two relevant vectors		**M1**	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result.		**M1**	
		Obtain $54.8°$ or 0.956 radians		**A1**	[4]
	(iii)	Either	Find at least one position of *C* by translating by appropriate multiple of direction vector **i** – 3**j** + 4**k** from *A* or *B*	**M1**	
			Obtain $(-3, 11, -20)$	**A1**	
			Obtain $(9, -25, 28)$	**A1**	
		Or	Form quadratic equation in λ by considering $BC^2 = 4AB^2$	**M1**	
			Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	**A1**	
			Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	**A1**	[3]
9	**(a)**	Either	Find *w* using conjugate of $1+3\mathrm{i}$	**M1**	
			Obtain $\dfrac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
			Square $x+\mathrm{i}y$ form to find w^2	**M1**	
			Obtain $w^2 = \dfrac{48-14\mathrm{i}}{25}$ and confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		Or 1	Find *w* using conjugate of $1+3\mathrm{i}$	**M1**	
			Obtain $\dfrac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
			Find modulus of *w* and hence of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Find argument of *w* and hence of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		Or 2	Square both sides to obtain $(-8+6\mathrm{i})w^2 = -12+16\mathrm{i}$	**B1**	
			Find w^2 using relevant conjugate	**M1**	
			Use correct process for finding modulus of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	

Or 3	Find modulus of LHS and RHS	**M1**	
	Find argument of LHS and RHS	**M1**	
	Obtain $\sqrt{10}\,e^{1.249i}\ w = \sqrt{20}\,e^{1.107i}$ or equivalent	**A1**	
	Obtain $w = \sqrt{2}\,e^{-0.1419i}$ or equivalent	**A1**	
	Use correct process for finding w^2	**M1**	
	Obtain 2 and –0.284 radians or $-16.3°$	**A1**	
Or 4	Find moduli of $2+4i$ and $1+3i$	**M1**	
	Obtain $\sqrt{20}$ and $\sqrt{10}$	**A1**	
	Obtain $\left\|w^2\right\| = 2$ correctly	**A1**	
	Find $\arg(2+4i)$ and $\arg(1+3i)$	**M1**	
	Use correct process for $\arg(w^2)$	**A1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 5	Let $w = a + ib$, form and solve simultaneous equations in *a* and *b*	**M1**	
	$a = \frac{7}{5}$ and $b = -\frac{1}{5}$	**A1**	
	Find modulus of *w* and hence of w^2	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 6	Find *w* using conjugate of $1+3i$	**M1**	
	Obtain $\frac{7-i}{5}$ or equivalent	**A1**	
	Use $\left\|w^2\right\| = w\bar{w}$	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	[6]

(b)	Draw circle with centre the origin and radius 5	**B1**	
	Draw straight line parallel to imaginary axis in correct position	**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
	Obtain $5e^{\pm\frac{1}{3}\pi i}$ or equivalents in required form	**A1**	[4]

10	**(i)**	State $\frac{dN}{dt} = k(N - 150)$	**B1**	[1]
	(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
		Obtain $k = 0.08$	**A1**	
		Separate variables and obtain general solution involving $\ln(N - 150)$	**M1***	
		Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$	**A1**√	
		Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
		Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent	**A1**	
		Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
	(iii)	Either Substitute $t = 15$ to find N or solve for t with $N = 2000$	**M1**	
		Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met	**A1**	[2]